

Estimating the Strategic Value of Long-Term Forward Purchase Contracts Using Auction Models

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ABSTRACT

We demonstrate how an auction model can be used in a traditional capital budgeting context to assign a value to the strategic advantage of long-term forward contracts. Research in the field of industrial organization has pointed to the danger of ex post opportunistic bargaining as a motivation for the use of forward contracts in natural resources and manufactured products, but no operational procedure exists for estimating the value secured by these contracts. Arbitrage methods for valuing forward contracts assume a competitive market in which the factors creating the bargaining problem and motivating the use of long-term contracts are not present. Use of the model is illustrated in the case of take-or-pay contracts for natural gas.

OVER THE LAST DECADE, much attention has been focused upon strategic factors and information problems that influence corporate financing decisions. The results of this research have been primarily suggestive—providing possible explanations of phenomena but not providing specific methods for incorporating the strategic factors into quantitative valuation techniques. Quantitative models of financial variables have been developed primarily for cases of perfect competition or similar special cases in which strategic factors are not central. In this paper we demonstrate how an important model developed in the literature on information economics can be used to estimate the value of strategic factors that must be incorporated in a traditional capital budgeting problem. Specifically, we apply a model of an auction to estimate the portion of a project's value that is secured to a firm through the use of long-term forward contracts for the product of the project's operation.

Long-term forward contracts are a typical element of financing for industrial projects in which large amounts of capital must be invested up front to develop production capacity and in which the market for the firm's output consists of a small number of buyers. Firms in such an industry often make the successful negotiation of forward purchase contracts a contingency upon which their decision to install capacity depends. By doing so they incur two advantages relative to firms that forego the use of forward contracts and choose instead first to

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install a given level of capacity and then to seek buyers for their products: i) they gain the information on demand that is revealed in a market price, and ii) they are likely to negotiate a higher sale price for their products since they can avoid the ex post bargaining problem. Long-term forward contracts, therefore, improve the efficiency of capital investment decisions both directly through the information they yield and indirectly because the firm appropriates the marginal value of its investment.

These two benefits to forward contracts have been discussed in two distinct sets of literature. The informational content of forward prices has been emphasized in the literature on rational expectations equilibria of a competitive economy, where by assumption all parties are able to utilize the information embedded in the publicly observable price.¹ The strategic importance of forward contracts has been stressed in the field of industrial organization, most notably in the theoretical work of Williamson (1975) and Klein, Crawford, and Alchian (1978) and in empirical work on the U.S. coal industry and electrical utilities by Joskow (1985).²

Our contribution to these two literatures is to make operational these theoretical insights in a capital budgeting context. We show how one can use an auction model to derive numerical estimates of the strategic value of forward contracts. The model integrates the value of avoiding ex post bargaining problems with the informational value of forward contracting. By developing a device for estimating the significance of these considerations for any given project, we provide a tool for making specific recommendations as to whether the long-term contracts should be used to help finance a particular capital project.

In the finance literature, long-term forward contracts have been interpreted primarily as instruments that shift an exogenous price risk from one party to another. The problem of valuing these contracts is therefore reduced to a valuation of the price risk. The techniques for analyzing the price risk component of forward contracts have been pioneered by Black (1976) and Cox, Ingersoll, and Ross (1981) and applied by Brennan and Schwartz (1985) to the valuation of long-term contracts for the output of a mine. In these papers, a stochastic process for the spot price of the commodity is postulated, and the pattern of forward prices that would be consistent with this process is derived. Prices are determined in competitive markets, and the relationship between spot and forward prices is determined by arbitrage. The producer cannot influence the information it obtains from the market by its decision to negotiate a forward contract; nor does it influence the expected future spot price by its decision to

¹ We analyze, in contrast, an imperfectly competitive market in which information is garnered only by those agents engaging in forward contracting.

² In both Williamson (1975) and Klein, Crawford, and Alchian (1978), the primary comparison is between long-term contracts on the one hand and vertical integration on the other. Long-term contracts are viewed in these papers as inherently unstable or incompletely enforceable so that they suffer from the ex post bargaining problem. Vertical integration is the alternative that permits the firm to appropriate the "quasi-rents" generated by its capacity decisions. In this paper, we have changed the labels for the relevant comparison. We represent an absence of contracting, i.e., waiting to use the forthcoming spot market, as the initial condition, and we analyze long-term contracts as well specified and enforceable alternatives that permit the firm to appropriate the "quasi-rents."

install capacity. In this arbitrage framework, the value to a seller of the forward contract relative to a decision to sell at the anticipated spot price is zero by construction: the equilibrium forward price is defined as that price which makes this value zero. Alternatively, for long-term contracts in which the delivery price is fixed and does not correspond to the equilibrium forward price, the value of the contract is the expected sum of the difference between this contracted price and the anticipated spot price. The arbitrage framework measures the value of the exogenous price risk that is shifted using these long-term contracts but assumes away the factors generating the ex post bargaining problem that motivates the use of these contracts in many cases.

The critical distinction between the model used in this paper and this arbitrage method is that the model used here involves imperfectly competitive markets. The producer's capacity decision and the timing for contract negotiations affect the price received: the spot price process cannot be taken as exogenous and the forward price is not determined by arbitrage vis-à-vis the anticipated spot price process. The value to the producer of forward contracts relative to spot transactions may therefore deviate from zero: the problem is to estimate how significant this deviation may be, i.e., to determine whether these strategic factors are significant. To accomplish this we explicitly recognize the impact that each player's decision will have on the outcome of the negotiations forward and spot. Unfortunately, to do this we need to abstract from the factors making a valuation of the price risk important and feasible.

In the next section we present an auction model and use it to define the strategic value of the forward contract. In Section II we use this model to estimate the strategic value of take-or-pay contracts for natural gas fields. Section III concludes.

I. An Auction Model of Expected Sales Revenue: Forward vs. Spot Prices

We consider the problem of the firm analyzing a traditional capital budgeting decision. The firm needs to estimate the cash flows to be received from constructing a plant or developing a natural resource deposit. A key problem is to estimate the prices at which the output can be sold. The firm can choose to sell its product under long-term forward contracts or under short-term or spot contracts, and it needs to make an estimate of the prices it can expect for both of these choices. The key difference between the forward and the spot market is that the firm negotiates the contract on the forward market prior to installing capacity, while it negotiates the spot sale after it has already installed its capacity: the two possible sequences of production and negotiation are illustrated in Figure 1.

Since both markets are imperfectly competitive, the installation of capacity substantially alters the strategic factors influencing the price. As the industrial organization literature has pointed out, given the same overall demand conditions, the seller should expect that, if it first installs capacity and then negotiates the sales, it will obtain a lower price on average than it would obtain in negotiations on forward contracts. The problem is to estimate this difference and its conse-

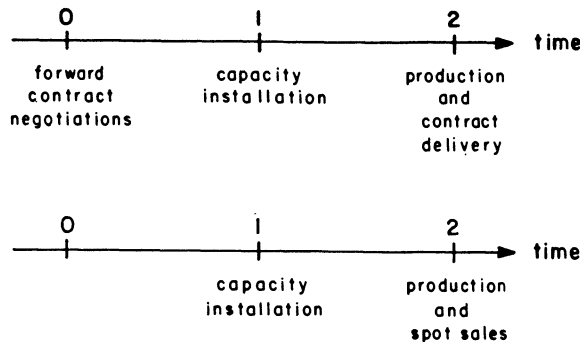


Figure 1. The time sequence of decisions under forward contracting and under production for spot sales.

quences for the ultimate value of the project. It is also important to estimate the value of the information on actual demand derived from the negotiations and used to determine whether the capacity should be installed at all.

The estimate of the value associated with these two factors is derived using a negotiation game played by the producer and its potential buyers in which the payoff structure and, therefore, the expected outcome of the game are altered by the decision to install capacity. When this negotiation game is played prior to the installation of capacity, it yields one probability distribution of likely outcomes—prices and quantity of sales under forward contracts. When this negotiation game is played subsequent to the installation of capacity, it yields another probability distribution of likely outcomes—spot prices and sales. The difference in expected profits from the two negotiation games is the estimate of the project value that is secured through the use of forward contracts as compared with spot sales.

A. *The Auction Model*

In this paper we use an auction model as an analog for the negotiation game. The auction model we use is a simple adaptation of the modified Vickrey auction analyzed in Harris and Raviv (1981). This auction is a variation on the English or competitive/uniform-price auction. Since the price in this auction is the same for all buyers and is set close to the reservation value of the marginal buyer, this auction is intuitively analogous to a competitive market. It also corresponds to our notion of a competitive market in the sense that, as the number of buyers in the auction grows, the results of the auction approach the results of a perfectly competitive market.

An auction model is a well defined analog for the negotiation process. The auction model we use has certain attractive properties relative to the class of possible rules for sale: 1) it is ex post efficient—that is, the buyers who value the product most receive it; 2) it is time consistent in the sense that the seller does not close the negotiations and leave capacity unused when there exist buyers willing to purchase the product at a price greater than marginal cost; and 3) among the class of selling mechanisms satisfying the time consistency property, it is the one that maximizes the seller's expected revenue. The auction model is

also advantageous for our purposes since the bidding rules and equilibrium strategies that yield the reduced-form results of the auction have been derived with explicit attention to the strategic relationship among the buyers and between the buyers and the seller. This is important since the critical factor distinguishing our analysis of forward contracts from the arbitrage method is that the markets in which the commodities are to be sold are imperfectly competitive and the seller's decision to install capacity affects the future equilibrium spot price through its effect on bargaining power in the future negotiations.

To define the model, we first explain the structure of the environment for which it is applicable. We assume that the Von Neumann-Morgenstern expected utility functions for the N potential buyers take the simple form in which each buyer desires up to a maximum of one unit of the commodity at any price below a given reservation price, r_i , so that $u_i(q, p) = r_i \min\{q, 1\} - pq$. The reservation price for each buyer is viewed by the seller as a random variable that may take on any of a finite set of reservation values, $\{R_1, \dots, R_l\}$, where $R_{j+1} = R_j +$, for $j = 1, \dots, l-1$. The probability distribution over this set of reservation values may be defined arbitrarily, but we will use for our examples the uniform distribution, $H(R_j) = 1/l$. The producer is characterized by the scalar parameter of its constant marginal cost function, c , for a quantity of production up to a maximum of Q .

In a Vickrey auction with a capacity of Q units and a constant marginal cost of production, there are two key rules determining the allocation of the output and payments that result from a given set of bids: 1) the Q buyers with the highest bids above the marginal cost of production receive the commodity—when fewer than Q bids are above the marginal cost of production, only those buyers bidding above marginal cost receive a unit, and some capacity is left unused; and 2) the price paid is equal either to the highest bid among those bids not accepted or to the marginal cost of production, whichever is greater. Buyers will bid based upon their realized reservation values and their assumptions about the probable bids of other buyers. In the symmetric Nash equilibrium to the Vickrey auction, each buyer's strategy is to submit a bid equal to its valuation. Consequently, the equilibrium outcome can be characterized as follows: 1) the Q buyers with the highest valuations receive the commodity—when fewer than Q buyers have reservation values above the marginal cost of production, only those buyers with reservation values above the marginal cost of production receive a unit; and 2) each buyer pays a price equal either to the highest valuation among those buyers not receiving the commodity or to the marginal cost of production, whichever is greater. Price is therefore determined by competition—when demand exceeds capacity, the price clears the market; otherwise, price is equal to marginal cost.

This characterization of the equilibrium outcome as a function of the realization of the N random reservation values can be formally defined as follows. Given a specific realization of the N -buyer reservation values, (r_1, \dots, r_N) , define (S_1, \dots, S_N) as the order statistics for the reservation values, so that $S_1 \geq S_2 \geq \dots \geq S_N$, $S_j \in \{r_1, \dots, r_N\}$. Then the price at which any units of the commodity are sold is

$$p = \begin{cases} S_{Q+1}, & S_{Q+1} > c, \\ c, & S_{Q+1} \leq c, \end{cases} \quad (1)$$

and the quantity allocated to each buyer is

$$q_i = \begin{cases} 1, & r_i > \max\{S_{Q+1}, c\}, \\ \gamma, & r_i = S_Q = S_{Q+1} \geq c, \\ 0, & r_i < \max\{S_Q, c\}, \end{cases} \quad (2)$$

where $\gamma = [Q - |\{S_j | S_j > S_Q\}|] / |\{S_j | S_j = S_Q\}|$, i.e., the share obtained by dividing the remaining capacity among the buyers with reservation values equal to S_Q .

In environments similar to ours but for which the range of possible reservation values is continuous, this simple Vickrey auction is optimal for the seller—that is, among the class of feasible, incentive-compatible, and time-consistent mechanisms, it maximizes revenue. However, for an environment such as ours with a discrete range of reservation values, we must alter slightly the price rule in order to guarantee revenue maximization—the modification and the reason for it are discussed in Harris and Raviv (1981):

$$p_m = \begin{cases} S_{Q+1}, & S_Q = S_{Q+1} \geq R_h, \\ S_{Q+1} + \delta A(Q+1), & S_Q > S_{Q+1} \geq R_h, \\ R_h, & R_h > S_{Q+1}. \end{cases} \quad (3)$$

For a set of values (A_1, \dots, A_{l-1}) as defined in Appendix 1, where $A(j) = A_i$ when $S_j = R_i$ and where $R_h = \min\{R_j | R_j \geq c\}$. This modified price function differs from the simple Vickrey auction price function defined in (1) in that, whenever the valuations of the lowest valued buyer receiving a unit and the highest valued buyer not receiving a unit are different, $S_Q > S_{Q+1}$, the price does not drop all the way to S_{Q+1} but is set at S_{Q+1} plus a fraction of the discrete interval between reservation values, that fraction being equal to $A(Q+1)$.

The auction rules specified in equations (2) and (3) differ from the modified Vickrey auction defined in Harris and Raviv (1981) only in terms of the minimum bid, R_h . This modification ensures the time consistency property and does not qualitatively change the strategic nature of the auction.³

For those readers wishing to familiarize themselves with the modified Vickrey auction, we have provided in Appendix 2 a display of the equilibrium outcomes of this auction model for a sample set of parameters.

³The optimal auction for a monopolist typically includes a minimum bid above the cost of production, and this is true in the modified Vickrey auction defined in Harris and Raviv. A minimum bid in an auction with multiple bidders is strategically analogous to a take-it-or-leave-it offer made by a seller bargaining with a single buyer. The imposition of a minimum bid implies that the seller has the power to commit itself to walk away from the auction if no potential buyer is willing to make a bid as high as this minimum; it implies that the seller is able to commit itself to refuse to sell at a lower price once it is revealed that none of the potential buyers is willing to bid the minimum. If the producer has no power to commit itself, then it is clear that a minimum bid above the cost of production cannot be a feature of the auction; a negotiation or selling strategy inclusive of a minimum bid above the marginal cost of production would not be a time-consistent strategy and therefore would not be credible. This fact would, of course, be anticipated by the buyers and impact their bidding strategies. The maximal minimum bid that is credible is the minimal reservation value greater than the marginal cost of production, and this is the minimum bid, R_h , which we utilize in our modified Vickrey auction as defined in (2) and (3). The logic of the proofs in Harris and Raviv can be directly applied to show that the auction defined by (2) and (3) maximizes revenue across the set of feasible and incentive-compatible selling mechanisms that satisfy time consistency.

B. The Comparison between Forward and Spot Sales

The difference between the prices negotiated in forward and spot sales follows from the sunk nature of the expenditures for capacity and the weakened bargaining position of the seller that results. The bargaining power lost by the seller enters into the final results of our modified Vickrey auction model through the definition of the minimum bid. The minimum forward price in the modified Vickrey auction is determined as the minimum reservation value greater than the constant marginal cost *inclusive of capital costs*:

$$R_{hf} = \min\{R_j \mid R_j \geq k + v\}, \tag{4}$$

where k is the scalar parameter of the assumed constant marginal capital cost function and v is the scalar parameter of the assumed constant operating cost function. Once capacity has been installed, however, i.e., for spot sales, the minimum price will be determined as the minimum reservation value greater than the marginal operating cost alone:

$$R_{hs} = \min\{R_j \mid R_j \geq v\}. \tag{5}$$

The price function for the forward sales, p_{mf} , is then the modified Vickrey price function from equation (3) where $R_h = R_{hf}$, and the price function for the spot sales, p_{ms} , is the modified Vickrey price function from equation (3) where $R_h = R_{hs}$. The quantity allocations to the buyers are accordingly the quantity allocations from equation (2) where for forward sales $c = k + v$ and $R_h = R_{hf}$ and for spot sales $c = v$ and $R_h = R_{hs}$.

The profit made by the producer from a forward sale of the capacity is then a simple function of the realization of the order statistics of the buyer reservation values:

$$\pi_f(S_1, \dots, S_N) = [p_{mf}(S_1, \dots, S_N) - (k + v)] \sum_{i=1}^N q_{if}(S_1, \dots, S_N), \tag{6}$$

and the expected profit from the decision to use a forward sale is

$$E(\pi_f) = \sum_{(S_1, \dots, S_N) \in \Psi} [\pi_f(S_1, \dots, S_N) g(S_1, \dots, S_N)], \tag{7}$$

where Ψ is the event space of possible combinations of order statistics (S_1, \dots, S_N) and $g(S_1, \dots, S_N)$ is the probability function defined on this space from the underlying probability distribution $H(\cdot)$ over the set of reservation values $\{R_1, \dots, R_n\}$.

For spot sales, the profit and expected profit functions are, respectively,

$$\pi_s(S_1, \dots, S_N) = ([p_{ms}(S_1, \dots, S_N) - v] \sum_{i=1}^N q_{is}(S_1, \dots, S_N)) - kQ \tag{8}$$

and

$$E(\pi_s) = \max\{0, \sum_{(S_1, \dots, S_N) \in \Psi} [\pi_s(S_1, \dots, S_N) g(S_1, \dots, S_N)]\}. \tag{9}$$

Note that, whenever the second argument in the maximum operator is negative, the producer can choose to forego any installation of capacity and thereby avoid any expectation of losses.

Our measure, Γ , of the project value secured to the producer by the use of forward contracts as compared with sales on the spot market can then be defined

as

$$\Gamma = E(\pi_f) - E(\pi_s). \quad (10)$$

The consequences of the weakened bargaining position of a seller with installed capacity are illustrated in Figures 2 and 3. In Figure 2 the probability distributions of forward and spot prices for a sample situation are displayed. The darker distribution represents the distribution of prices that are anticipated as a consequence of forward negotiations. The lighter distribution represents the distribution of prices that are anticipated as a consequence of spot sales. In Figure 3 two probability distributions over unit profits inclusive of unit capital charges are displayed. The darker distribution represents the unit profits from forward contracting, and the lighter distribution represents the unit profits from spot contracting. The literature on opportunistic bargaining yielded the insight that the distributions for spot sales would be shifted to the left. What was missing, and what the use of an auction model promises to give us, is a consistent method for assessing the magnitude of this shift for a well specified environment. The model also integrates with this measure the informational value of forward contracts. In Appendix 2 we provide a numerical example using our auction model to compare the forward and spot price distributions and to calculate Γ ; a review of the example will aid in understanding the determinants of Γ .

We close this section by formalizing the argument that the ex post bargaining problem and the informational value of forward contracting imply a higher level of profits from forward contracts than from spot sales, i.e., that $\Gamma \geq 0$.

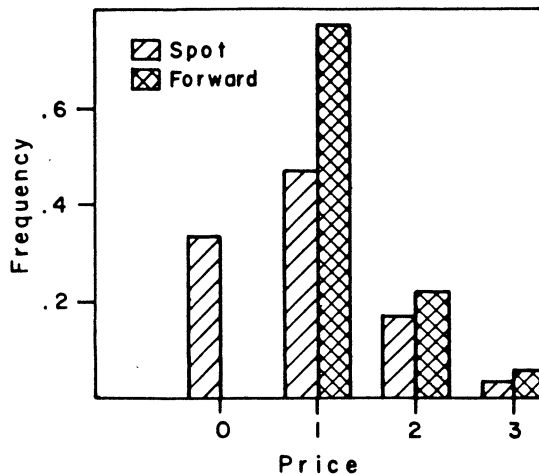


Figure 2. A sample probability distribution of forward and spot prices. The darker set of bars represents the anticipated price distribution from forward sales, i.e., using the auction rules described in equations (3) and (4), for the parameters given in Example 2 of Appendix 2. The lighter set of bars represents the anticipated price distribution for spot sales, i.e., using the auction rules described in equations (3) and (5), for the same parameters. To keep the two distributions visually simple and comparable, the probability weights assigned to the modified Vickrey auction prices $S_{Q+1} + \delta A(Q + 1)$ have been reassigned to the prices S_Q and S_{Q+1} while maintaining the same expected value.

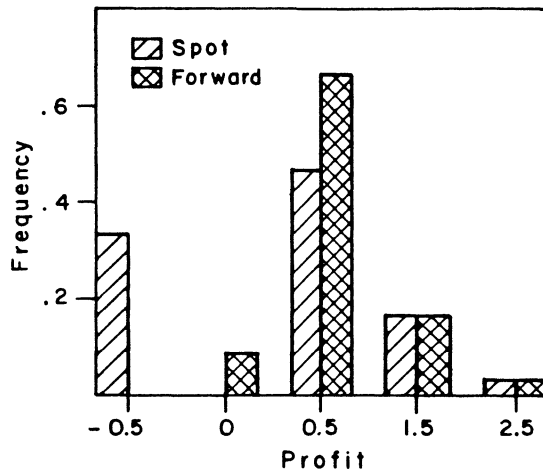


Figure 3. A sample probability distribution of forward and spot unit profits. The darker set of bars represents the anticipated profit distribution from forward sales, i.e., using the auction rules summarized in equation (6), for the parameters given in Example 2 of Appendix 2. The lighter set of bars represents the anticipated profit distribution for spot sales, i.e., using the auction rules summarized in equation (8), for the same parameters.

THEOREM 1: $\forall \{N, Q < N, (R_1, \dots, R_l), k, v, H(\cdot)\}, \Gamma \geq 0$, and
 $\exists \{N, Q < N, (R_1, \dots, R_l), k, v, H(\cdot)\}$ such that $\Gamma > 0$.

Proof: The first part of the proposition holds trivially when $R_{hs} = R_{hf}$, and the second holds only when $R_{hs} < R_{hf}$; we therefore assume k, v , and R such that this strict inequality holds. To establish the proposition, we examine the possible cases for realizations of the two parameters S_Q and S_{Q+1} that determine the prices and quantities sold in the forward and spot auctions. *Case 1:* $R_{hs} < R_{hf} \leq S_{Q+1}$. Then price and quantity allocations are identical for forward and spot auctions. *Case 2:* $[R_{hs} \leq S_{Q+1} < R_{hf}]$ and $[S_{Q+1} < S_Q]$ and $[k + v \leq S_{Q+1} + \delta A(Q + 1)]$. Then the following series of inequalities holds: $R_{hs} \leq S_{Q+1} < k + v \leq S_{Q+1} + \delta A(Q + 1) = p_{ms} < S_{Q+1} + \delta = R_{hf} = p_{mf} \leq S_Q$. Since $k + v \leq p_{ms} < p_{mf}$ and $\Sigma q_{is} = \Sigma q_{if}$, we have $0 \leq \pi_s < \pi_f$. *Case 3:* $[R_{hs} \leq S_{Q+1} < R_{hf}]$ and $[S_{Q+1} < S_Q]$ and $[S_{Q+1} + \delta A(Q + 1) < k + v]$. Then the following series of inequalities holds: $R_{hs} \leq S_{Q+1} < S_{Q+1} + \delta A(Q + 1) = p_{ms} < k + v \leq R_{hf} = p_{mf}$. Since $p_{ms} < k + v \leq p_{mf}$, we have $\pi_s < 0 \leq \pi_f$. *Case 4:* $[R_{hs} \leq S_{Q+1} < R_{hf}]$ and $[S_{Q+1} = S_Q]$. Then the following series of inequalities holds: $p_{ms} = S_{Q+1} = S_Q < k + v \leq R_{hf} = p_{mf}$, and, therefore, $\pi_s < 0 \leq \pi_f$. *Case 5:* $[S_{Q+1} < R_{hs} < R_{hf}]$. Then $p_{ms} = R_{hs} < k + v \leq R_{hf} = p_{mf}$ and $\pi_s < 0 \leq \pi_f$. These exhaust the possible cases. Since in every case $\pi_s \leq \pi_f$, it follows that $\Gamma \geq 0$. To establish the second part of the theorem, it is sufficient to choose the parameters such that π_f is strictly greater than zero and such that one of cases 3–5 occurs with positive probability. Q.E.D.

Remarks: The intuition behind the main result is brought out in the proof as follows. In case 1, where competition is great, the price and quantity allocations are, therefore, expected profits from spot and forward sales are identical. In all other cases, the problem of selling to an imperfectly competitive market is central.

This imperfect competition implies that the negotiated price on the spot market is weakly lower than the price on the forward contract; in cases 2–5, the proof establishes that the unit operating profits are weakly greater for forward sales than they are for spot sales. This is true since the minimum forward price is greater than the minimum spot price. Of course, there will be some events in which the producers would sell a larger quantity spot than they would forward—this is possible in cases 3–5. In these cases, however, the fully allocated unit profits from selling spot are negative. Although it is optimal to make the extra spot sales at the low price given that the capital costs are sunk, the initial decision to expand the capacity is *ex post* regrettable. With forward contracting, the decision to expand capacity in these events is avoidable.⁴

C. Generalizations

Implicit in our choice of a particular auction model are a large number of simplifying assumptions that restrict the applicability of the model. Some of the assumptions can be relaxed in a direct manner. The important contribution of this paper is in showing how existing game theoretic models of negotiations can be used to estimate the difference between the profitability of contracting before and after capital investments have been sunk. The particular assumptions of the game model used here are not central to this point. Therefore, where there exist other models incorporating a different set of assumptions, these may be applied in a directly analogous manner. If, for example, the buyers of the commodity were to be considered averse to the risk of obtaining the commodity through this seller, then a different but well known auction, probably a discriminatory auction, would be optimal (Matthews, 1983). Alternatively, in a particular capital budgeting context, the competition between the small number of other potential sellers of the commodity may be central to the negotiations over forward contracts. In this case a model of bilateral auctions would be necessary (Wilson, 1982). It is also likely that in many cases the reservation value of the buyers should not be modeled as independent random variables but should be modeled as correlated or affiliated random variables. In this case auction theory instructs us that an auction different from the modified Vickrey auction used here would be the optimal selling mechanism and that the estimates of the value of contracting would be different (Weber, 1983). None of these changes in assumptions requires a significant modification of the ideas in this paper. The question centers exclusively upon whether an appropriate model has been developed for the given context, and in many cases such a model already exists. Unfortunately, for some apparently simple modifications of the assumptions, the optimal selling mechanism has not been discovered or analyzed. For example, several state governments have instituted various auction procedures by which public utilities will accept

⁴ This paper focuses exclusively upon the positive value to forward contracts and therefore Γ is always positive. Presumably, these contracts impose constraints on future transactions which are costly and which might make Γ negative in some cases. A full treatment of the problem would measure both the strategic costs and benefits for an optimally designed contract. We consider our estimation of the benefits to be one element of this process and are pursuing research on modeling the strategic costs to long-term contracts and the relation between the benefits and the costs.

contracts for congenerated power (Rothkopf et al., 1987). A precise solution to the optimal auction procedure for awarding the contracts has not yet been derived, precisely because the problem involves modifying the assumptions of this paper about the slope of the demand curves of the buyers in such a fashion that the optimal auction mechanism is not readily derived. In cases such as this one, then, the further application of the techniques presented here is dependent upon advances in game theory.

There is a second set of assumptions that is not so easily or directly modified without at the same time delving further into the problem of contracting. For example, in our model we have collapsed the negotiations over a contract that covers twenty years of deliveries into negotiations over a single parameter—the average price to be paid over the life of the contract. Similarly, we have collapsed the expectations about all future spot negotiations into one parameter—the average price to be paid over all future spot purchases. Correspondingly, we have represented the uncertainty characterizing each buyer as a single parameter probability distribution. We have assumed that the same distribution characterizes a buyer's reservation value for the average forward contract price and a buyer's reservation value for the average price to be paid over all future spot purchases. In our simplified model, these are correctly viewed as identical. However, a more complicated model could more accurately reflect the complications of actual forward contract negotiations and the alternatives facing a buyer vis-à-vis the future spot markets. For example, each buyer might be more accurately characterized by an increasing degree of uncertainty over time. In a subset of cases, this more complicated environment collapses mathematically to the model that we have presented here, but in some cases in which flexibility is important and in which the forward contract and the future spot markets afford different amounts of flexibility, the model presented here is not capable of accounting for these differences. It is to questions of this sort that future research on the analysis and valuation of forward contracts must be directed.

II. Natural Gas Take-or-Pay Contracts

As an example of the application of this model for the estimation of the value to a producer of a forward contract, we use the take-or-pay contracts that are common in the natural gas industry. Typically, once a natural gas field has been discovered, and simultaneously with the decision to develop the field, the producer negotiates with prospective buyers long-term contracts under which a customer commits itself to pay each year for a given quantity of the gas. The buyer must pay for the gas to which it has committed itself whether or not it actually takes the gas—hence the name take-or-pay. These commitments usually run for a duration of fifteen to twenty years. In many contracts, the exact quantity to be taken and the price to be paid in any given year may be described either by a complex formula or by more indefinite contingencies including potential regulatory changes.

The “ex post bargaining problem” is commonly recognized as one primary motivation for the use of take-or-pay contracts in the natural gas industry. Its importance has historically been so great that almost all gas fields were financed

using take-or-pay contracts that committed nearly 100 percent capacity of the available gas. Flexibility consisted exclusively of the right to transfer delivery of a small portion of contracted quantities across a minimal number of years. Little interest existed, therefore, in precisely assessing the significance of the strategic value of the long-term contracts.

The situation has changed dramatically in recent years. Short-term markets in natural gas now coordinate a significant portion of deliveries. Producers in the United States and Canada have found themselves studying the development of a field that appears by all traditional capital budgeting rules to be economically justifiable but for which they are unable to negotiate successfully the traditional level of "take" commitments on the part of buyers. Similar problems have arisen for producers targeting the western European market. Producers have therefore raised the question of whether the traditional rules of thumb for the minimal long-term contract quantity of gas needed to justify the installation of capacity should be changed. The problem of assessing the significance of the strategic value of long-term contracts to a particular developer has therefore become important.

In this section, we demonstrate the use of the auction model to assess the importance of these contracts for four natural gas fields. To illustrate how different features of the market impact the strategic value of the forward contracts, we make two comparisons using the auction model. The first comparison focuses upon the number of potential customers. We analyze the strategic value of forward contracts for the Venture natural gas field in eastern Canada and compare it with the strategic value of forward contracts for the development of a field in Alberta. The market for the gas from the Venture field consists of a very small set of users in New England. The gas from the Albertan field can be routed to a larger number of users in the midwestern and west coast United States. We will see that the model estimates the strategic value of forward contracts to be significantly greater for the Venture field than for the Albertan field. This occurs because the competition among the many potential buyers for the Albertan gas makes the capacity decision less important for the seller's bargaining power.

The second comparison focuses upon the size of the initial capital expenditures necessary to make the gas deliverable. We analyze the Troll field in Norway and the Soviet gas field in Urengoi. The Troll field reportedly requires relatively low capital expenditures. The Soviet gas requires large expenditures on pipelines and therefore larger initial capital expenditures to make the gas deliverable. The model calculates the strategic value of forward contracts to be relatively modest for the Troll field, while for the Soviet gas field the value of the forward contracts is significant.

A. The Number of Buyers: The Venture Field vs. Albertan Fields

To assess the strategic value of a take-or-pay contract for a field, we first need to determine the parameter values for the variables of our model. These include the field characteristics: (i) the size of the field, (ii) the per-unit capital costs, and (iii) the per-unit operating costs. These also include the market or buyer

characteristics: (iv) the number of buyers and (v) the range of possible reservation values for each buyer. We will depend upon figures for the Venture gas field that are taken from Adelman et al. (1985) and use this data to choose the values for our parameters.⁵

Field characteristics. The Venture field has total reserves of 2.36 Tcf (trillion cubic feet) and will be operated at a level of 116.8 Bcf (billion cubic feet) per year for a period of twenty years. We will analyze the value of contracts on an annual basis, and, therefore, the total capacity used in the auction model will be this annual capacity. The capital expenditures necessary to develop the field are \$1837.5 million. These expenditures are amortized over the life of the field and the quantity to be delivered each year (for this example and throughout the remainder of the paper we use a 12 percent discount rate), yielding a capital cost of \$2.66/Mcf (\$/thousand cubic feet). This will be the constant marginal capital cost figure used in the model. The operating costs are \$75 million/year; the field, however, will produce associated liquids the sale of which will approximately equal the operating costs, and, therefore, we will set the operating costs per Mcf at zero.

Market/buyer characteristics. The gas from the Venture field will be sold in the northeastern U.S. market, where there are a relatively small number of large buyers. Current sales by Canadian producers to this market are less than double the capacity of the Venture field. We model the negotiations as taking place between the field developer and three buyers, each of which would utilize the full annual capacity of the field; negotiations are therefore modeled as an auction of a single unit—116.8 Bcf/yr of gas—to three bidders. The range of prices to which these buyers might agree in a contract will be determined by the alternative sources of supply that are available to these buyers. Additional western Canadian gas might be available in the near future in this market at a price of approximately \$2.90/Mcf at the border and at \$3.15 or \$3.40/Mcf at the border in the next decade. Additional gas from Louisiana would be available, delivered in Boston at \$4.02/Mcf. Current consumption levels could perhaps be supplied from Louisiana at as low as \$2.80/Mcf delivered. Transportation costs from the Venture field to the border and to Boston could be as high as 60 and 90 cents per Mcf, respectively, although figures of 30 and 50 cents are perhaps more realistic. The range of wellhead prices that customers in the northeastern U.S. are likely to accept may therefore lie between \$2.30 and \$3.52/Mcf or could be as low as \$2.00 to \$3.10/Mcf. We use the former pair of prices to bound the range of buyer reservation values for our sample calculations in this paper.

These parameter values and the model calculations for expected profit from forward and spot contracting are displayed in Table I.

The results of the model run for the Venture field can be interpreted as follows. If the developer of the Venture field enters into negotiations for forward commitments on the purchase of gas prior to the expenditure for development and pipeline construction, then the expected annual profit is \$46 million. That totals

⁵ The numbers and examples used in this paper are meant to illustrate the use of the auction model and are not meant as definitive analyses of the chosen projects, and we, therefore, do not detail the derivation of the data.

Table I
**The Strategic Value of Forward Contracts for
the Venture Gas Field**

Net Present Values are calculated as follows. The values for the model inputs are used to calculate the expected annual profits to sales, forward and spot, according to equations (5)–(8) of the model. Annual profits are transformed into project NPVs using an annuity formula based upon the specified life of the project and a real discount rate of 12%.

Model Inputs:		Outputs, NPV:	
Field Size	116.8 Bcf/yr; ^a	Forward Sales	\$344 million
	20 yr	Spot Sales	\$307 million
Capital Cost	\$2.66/Mcf ^b	Difference (Γ)	\$37 million
Operating Cost	\$0.00/Mcf	% Change	–10.8
No. of Buyers	3		
No. of Sale	1		
Units			
Max. Res. Value	\$3.52/Mcf		
Min. Res. Value	\$2.30/Mcf ^c		

^a Bcf: billion cubic feet.

^b Mcf: thousand cubic feet.

^c The range of reservation values is defined as the six values equally spaced between the maximum and the minimum reservation values.

\$344 million NPV over the twenty-year life of the project. If the developer were to install the capacity and subsequently attempt to sell the output “spot,” then the expected annual profit calculated by the model is \$41 million, or \$307 million NPV over the life of the project.

These averages incorporate the possibility that in the negotiations the developer may find no buyer willing to commit itself to purchase the gas at the minimum price of \$2.66/Mcf; the probability of this event is 3.7 percent. In the case of forward contracting, the developer would cancel the project and incur no losses. In the case of spot sales, the developer would have already made the capital expenditures and would sell the output at a price below the fully allocated cost, incurring a per-unit loss. For an additional set of cases with a probability of 20 percent, the spot price covers capital and allocated costs, but it is still less than the forward price by about \$0.15/Mcf. These two factors combine to make the profits expected from a strategy of development and spot sales \$5 million less annually than the profits from a strategy of forward negotiations and development contingent on their outcome. This is a total net present value loss over the life of the project of \$37 million, or 10 percent of the NPV. This is the portion of the project NPV that is endangered in spot negotiations by the ex post opportunism of the buyers; this is the portion of project NPV that is secured by means of forward contracts.

Albertan field characteristics. Data on the Albertan fields are also taken from Adelman et al. (1985). While the Venture data were derived from a field-specific source, the Albertan field data are based upon typical costs for a class of fields

in Alberta. We will model a field with a twenty-year annual production capacity of 50 Bcf. The capital expenditures necessary to develop this field amount to \$300 million over a three-year period, or an amortized expenditure based upon the planned rate of depletion of \$1.01/Mcf. The operating expenditures necessary are \$0.45/Mcf.

Market/buyer characteristics. The key difference between the Albertan field and the Venture field is that the Albertan field can be connected into various pipeline networks, each of which in turn serves a broader number of large customers. Access is available both to the dense set of pipelines serving the U.S. midwest and to pipelines serving the U.S. west coast. For illustration we will model the problem as negotiations involving five large buyers, each of which can consume the full output of the field; hence, the developer is negotiating to sell a single unit to one of five buyers. The range of reservation prices that we will use for each buyer is \$1.00–4.00/Mcf, a range which generates an expected price of \$3.22/Mcf. This price is comparable to that calculated in Adelman et al. (1985) as a likely scenario for exports from Alberta to the midwestern and western U.S.

These parameter values and the model calculations for expected profit from forward and spot contracting are displayed in Table II. Displays of the probability distributions of forward and spot prices and of the probability distributions of forward and spot profits generated by the auction model of negotiations for the Venture and Albertan fields appear in Figures 4 and 5.

The results of the model run for an Albertan field can be interpreted as follows. If the developer of the Albertan field enters into negotiations for forward commitments on the purchase of gas prior to the expenditure for development

Table II
The Strategic Value of Forward Contracts for the Albertan Gas Field

Net Present Values are calculated as follows. The values for the model inputs are used to calculate the expected annual profits to sales, forward and spot, according to equations (5)–(8) of the model. Annual profits are transformed into project NPVs using an annuity formula based upon the specified life of the project and a real discount rate of 12%.

Model Inputs:		Outputs, NPV:	
Field Size	50 Bcf/yr; ^a	Forward Sales	\$695 million
	20 yr	Spot Sales	\$695 million
Capital Cost	\$1.01/Mcf ^b	Difference (Γ)	\$0 million
Operating Cost	\$0.45/Mcf	% Change	–0.2
No. of Buyers	5		
No. of Sale	1		
Units			
Max. Res. Value	\$4.00/Mcf		
Min. Res. Value	\$1.00/Mcf ^c		

^a Bcf: billion cubic feet.

^b Mcf: thousand cubic feet.

^c The range of reservation values is defined as the six values equally spaced between the maximum and the minimum reservation values.

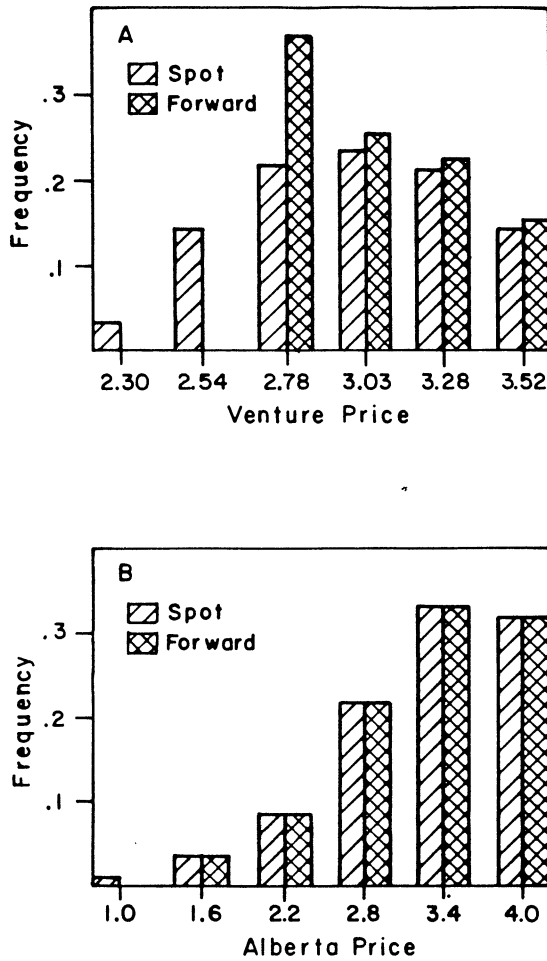


Figure 4. Probability distribution of forward and spot prices for the Venture and for the Albertan gas fields. The darker set of bars in Panel A is the anticipated distribution of the forward prices that could be negotiated for the Venture gas field, and the lighter set of bars is the anticipated distribution of spot sale prices that could be negotiated for the Venture gas. Panel B displays the same pair of distributions for the Albertan field. In the case of Venture, the forward price distribution is clearly shifted rightward or stochastically dominates the spot distribution. One can see in this shift of the distribution the strategic value to forward contracts. In the case of Alberta, the forward and spot distributions are almost identical, and, hence, one can see that the strategic value to forward contracts in the case of Albertan gas is virtually zero.

and pipeline construction, then the model calculates the expected annual profits to be \$186 million, or \$1.39 billion NPV, over the life of the project. If the developer were to install the capacity and then attempt to sell the output “spot,” then the expected annual profits calculated by the model would also be about \$186 million; the exact difference is two tenths of a percent of the annual profits. This is the portion of the project NPV that is endangered by the ex post opportunism of the buyers and that is secured by means of forward contracts.

A comparison of the results displayed in Tables I and II illustrates the difference between the strategic values of the forward contracts for the Venture field and for the Albertan field. While the forward contracts secure for the producer more than 10 percent of the NPV of developing the Venture field, they are virtually irrelevant to the developer of a field in Alberta.

The forward contracts offer little strategic value in Alberta since with five bidders the probability is small that there are not at least two buyers with reservation prices above the marginal costs inclusive of capital charges, and, therefore, there is little probability that competition among the buyers will be absent, leaving the seller dependent upon its bargaining power for ensuring a price sufficient to cover the capital and operating expenses. This can be seen in a comparison of the anticipated probability distributions of forward and spot prices and profits for the Venture field with the anticipated distributions for the Alberta field, as displayed in Figures 4 and 5. The average profit from forward contracting for an Albertan field incorporates the possibility that in the negotiations the developer may find no buyer willing to commit itself to purchasing the gas at the minimum price of \$1.46/Mcf; the probability of this event is a mere 0.01 percent (contrasted with 3.7 percent for Venture), and the probability of all

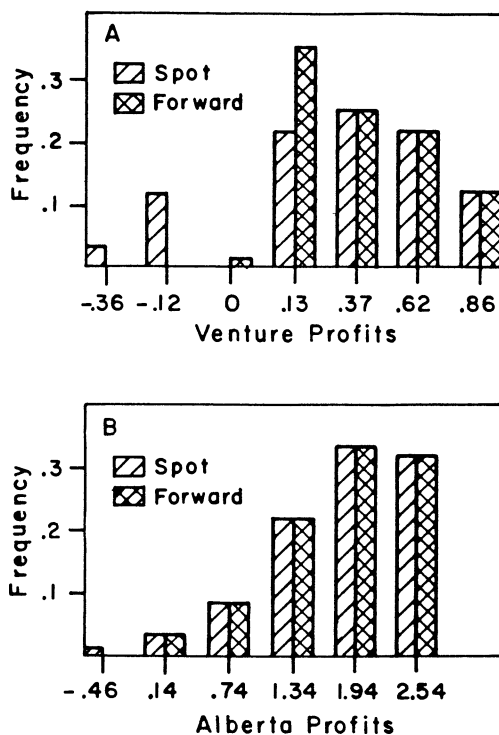


Figure 5. Probability distribution of forward and spot unit profits for the Venture and for the Albertan gas fields. The darker set of bars in Panel A is the anticipated distribution of unit profits to be earned on forward sales negotiated for the Venture gas field, and the lighter set of bars is the anticipated distribution of unit profits to be earned on spot sales of the Venture gas. Panel B displays the same pair of distributions for the Albertan field.

events in which the spot price is less than the forward price is only 0.24 percent (contrasted with 20 percent for Venture).

Assuming that we have captured the central motivation for the forward contracts, it would appear from our results that the gradual development of a "thicker" market in natural gas in the midwestern and western United States has significantly reduced for some fields the necessity of utilizing the strong take-or-pay contracts that have been common for the past several decades. Several persons in this industry have made such assertions, arguing that an increased reliance upon short-term sales is possible; our model calculations support this claim. It is important to note, however, that this possibility is restricted to particular areas: it would not, for example, be possible for gas to be marketed on this basis in New England, and, as we shall see, it is not possible for several other suppliers and markets in the world. The fact that these changes in the importance of contracts impact specific markets differentially is often ignored. While the source of the difference between the Venture and the Alberta projects is the number of buyers available to each, since the projects differ in other respects as well it is difficult to develop from their comparison an accurate sense of how sensitive the strategic value calculated by our model is to a change in the number of buyers. We report, therefore, in Table III the results of a comparative statics test on the Venture field in which the number of buyers is varied between two and six. The estimated difference between forward contracting and spot sales is \$118 million, or 54 percent of the project NPV, for the case of two buyers and falls to just over \$1 million, or less than 1 percent of the project NPV, in the case of six buyers.⁶

A second important comparative statics result focuses upon changing the probability distribution over buyer reservation values. Here many comparisons could be made. One such interesting comparison is to check the model results for a probability distribution that has smaller tails than the uniform distribution. In Table IV, therefore, we report the results of using our modified Vickrey auction, substituting a binomial distribution on the buyer reservation values. Since the ranges of the reservation values remain identical, a binomial distribution with a parameter of $x = 0.5$ and, therefore, with a mean identical to that of the uniform distribution has a variance much less than that of the uniform distribution; for the Venture field parameters, the variance of the uniform distribution of reservation prices is 0.17, while the variance on the centered binomial distribution is 0.07. Consequently, the estimate of the strategic value of forward contracting is significantly lower for this binomial distribution than for the uniform distribution—\$9 million, or 3 percent of the project NPV, as compared with \$37 million, or 11 percent of the project NPV. However, it should also be noted that, if the binomial distribution is only slightly skewed downward with a parameter of $x =$

⁶ The reader should interpret these results carefully, however. Although it appears as if only the number of buyers is being changed, in fact this change implies a particular assumption about changing aggregate demands. Simply multiplying the number of buyers in this model implies shifting the aggregate demand curve outward: it is not the same thing as keeping the aggregate demand constant and imagining the source of this demand to be divided among a larger number of buyers. Therefore, as the reader will note, the total profits from forward contracting are also increasing as the number of buyers is increased.

Table III
Comparative Statics: Variation in the Number of Buyers

Values reported are the expected profits for the Venture gas field calculated using the full set of input data displayed in Table I, except for the number of buyers, which is varied as displayed here.

Number of Buyers:	2	3	4	5	6
Forward Sales (\$ million)	218	344	441	512	564
Spot Sales (\$ million)	100	307	429	508	563
Difference (Γ) (\$ million)	118	37	12	4	1
% Change	-54	-11	-3	-1	-0

Table IV
Comparative Statics: Variation in the Probability Distribution

Values reported are the expected profits for the Venture gas field calculated using the full set of input data displayed in Table I. Expected profits from forward sales are calculated according to equation (7), and expected profits from spot sales are calculated according to equation (9). The underlying probability distributions over buyer reservation values are binomial distributions described by the parameter p as shown in the first line of the table. The binomial distribution with $p = 0.2$ is skewed toward the maximum reservation value of \$3.52/Mcf, while the distribution with $p = 0.8$ is skewed toward the minimum reservation value of \$2.30/Mcf.

Parameter Value, p :	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Forward Sales (\$ million)	611	509	405	303	212	137	76
Spot Sales (\$ million)	611	509	402	294	184	74	0
Difference (Γ) (\$ million)	0	0	3	9	28	63	76
% Change	0	0	-1	-3	-13	-46	-100

0.6, then the estimate of the strategic value of forward contracts rises to \$28 million, or 13 percent of the project NPV.⁷

B. The Proportion of Capital Costs: Troll (Norway) vs. Urengoi (Soviet) Gas

Data on Norway's offshore Troll natural gas field are taken from Adelman et al. (1986). Total volume available from the Troll field is 14.2 Tcf, or 610 Bcf per year for each of twenty-three years. Total capital expenditures for development will be \$3.2 billion, or \$0.67/Mcf amortized over the schedule of production. Expenditures for pipeline construction will be another \$2.56 billion—\$0.62/Mcf. Per-unit capital costs for the Troll field, therefore, amount to \$1.29/Mcf. Operating expenses for the field will be \$0.23/Mcf; operating expenses for the pipeline will be \$0.18/Mcf. Total operating expenditures are, therefore, \$0.41/Mcf.⁸

⁷ An important caveat to this experiment must be stated. The equations for the reduced-form results of our auction, and in particular the A values that appear in equation (3) and that are defined in Appendix 1, are derived under the assumption of a uniform distribution. The exact values should be derived anew for each binomial distribution in order to give correct estimates. The estimates given in Table IV do not incorporate this adjustment and so can only be viewed as roughly illustrative.

⁸ The data for Troll used here originated in early published reports and may be relatively optimistic compared to later estimates.

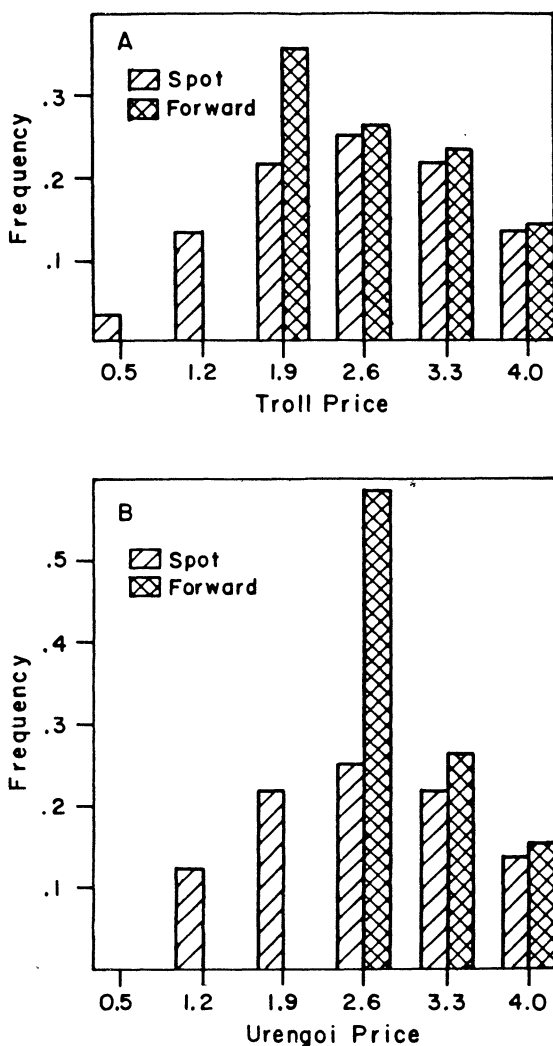


Figure 6. Probability distribution of forward and spot prices for the Troll and for the Urengoi gas fields. The darker set of bars in Panel A is the anticipated distribution of forward prices that could be negotiated for the Troll gas field, and the lighter set of bars is the anticipated distribution for spot sale prices that could be negotiated for the Troll gas. Panel B displays the same pair of distributions for the Urengoi field. In the case of Troll, the forward price distribution is moderately shifted rightward relative to the spot distribution, and one can see in this shift the moderate strategic value to forward contracts. In the case of Urengoi, the forward distribution more radically dominates the spot distribution, and, hence, the strategic value to forward contracts is dramatically greater.

The gas from the Troll field will be piped into the western European market, with France and the Federal Republic of Germany being the main buyers and potentially displacing gas to Italy. We will model the gas as being sold to three buyers. Each buyer is assumed to be able to purchase completely the scheduled annual output of 610 Bcf. The range of reservation values is based upon the demand profile provided in Adelman et al. (1986): \$0.50–4.00/Mcf.

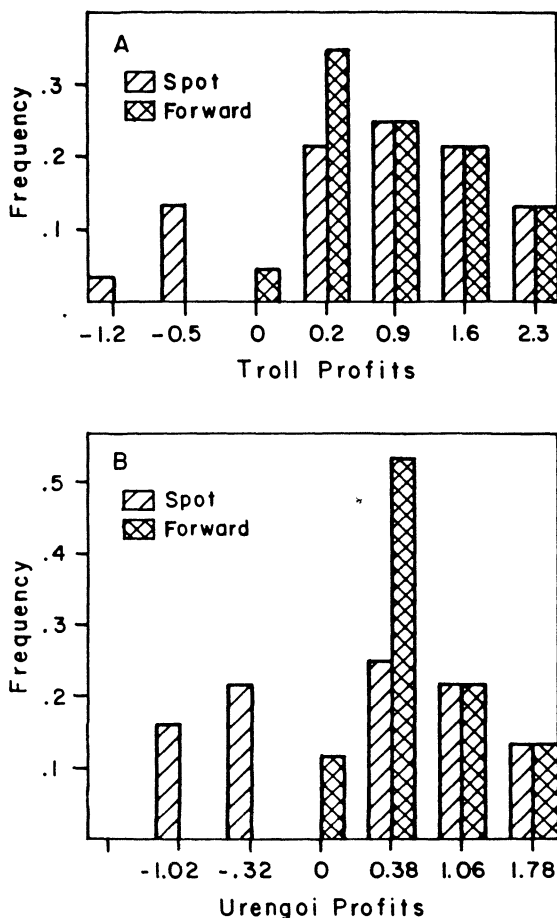


Figure 7. Probability distribution of forward and spot unit profits for the Troll and for the Urengoi gas fields. The darker set of bars in Panel A is the anticipated distribution of unit profits to be earned on forward sales negotiated for the Troll gas field, and the lighter set of bars is the anticipated distribution of unit profits to be earned on spot sales of the Troll gas. Panel B displays the same pair of distributions for the Urengoi field.

In the case of the Soviet Urengoi gas field, we will examine an expansion of production and transportation capacity of 1412 Bcf/yr, producing for twenty years. Per-unit capital costs for the Urengoi field are \$1.59/Mcf. Per-unit operating costs are \$0.63/Mcf. The Soviet gas will be sold to the same markets to which the Troll gas will be sold. We, therefore, model it as facing the same number of buyers and the same set of reservation values.

The parameter values and model calculations for these two fields are displayed in Table V.

A comparison of the results for the Troll field with the results for the Urengoi field show the impact of the higher capital costs of the Urengoi field on the strategic value of the forward contracts. Given the same range of reservation values and the same number of buyers, the probability that there exist at least

Table V
The Strategic Value of Forward Contracts for the Troll & Urengoi Gas Fields

Net Present Values are calculated as follows. The values for the model inputs are used to calculate the expected annual profits to sales, forward and spot, according to equations (5)–(8) of the model. Annual profits are transformed into project NPVs using an annuity formula based upon the specified life of the project and a real discount rate of 12%.

Model Inputs, Troll:		Outputs, NPV:	
Field Size	610 Bcf/yr; ^a	Forward Sales	\$4,571 million
	23 yr	Spot Sales	\$3,965 million
Capital Cost	\$1.29/Mcf ^b	Difference (Γ)	\$606 million
Operating Cost	\$0.41/Mcf	% Change	-13.3
No. of Buyers	3		
No. of Sale Units	1		
Max. Res. Value	\$4.00/Mcf		
Min. Res. Value	\$0.50/Mcf ^c		

Model Inputs, Urengoi:		Outputs, NPV:	
Field Size	1412 Bcf/yr;	Forward Sales	\$7,202 million
	20 yr	Spot Sales	\$3,572 million
Capital Cost	\$1.59/Mcf	Difference (Γ)	\$3,630 million
Operating Cost	\$0.63/Mcf	% Change	-50.4
No. of Buyers	3		
No. of Sale Units	1		
Max. Res. Value	\$4.00/Mcf		
Min. Res. Value	\$0.50/Mcf		

^a Bcf: billion cubic feet.

^b Mcf: thousand cubic feet.

^c The range of reservation values is defined as the six values equally spaced between the maximum and the minimum reservation values.

two buyers that value the resource at a price above the capital expenditures is greater for the field with the lower capital expenditures, Troll, than for the field with the higher capital costs. While it is immediate that the field with the lower costs per unit of production shows a higher profit margin when facing identical sets of buyers, our concern here is not with the absolute profit level but with the percentage of the margin that is secured via contracts. The auction model allows us to identify this value.

Again, one can see these results in a comparison of the forward and spot price and profit distributions for the two fields, as displayed in Figures 6 and 7.

It is also interesting to make a comparative statics analysis of the impact of capital costs on the strategic value using one project. In Table VI, we display the results of such an analysis for the Troll project. We hold constant all of the parameters of the Troll project except for the capital cost figure and examine how responsive the estimate of the strategic values is to changes in the capital cost. As one can read in Table VI, the estimate of the strategic value of forward contracting rises from a low of \$89 million, or 1 percent of project NPV, when the capital costs are only \$0.69/Mcf to a high of \$2,447 million, or 100 percent of project NPV, when the capital costs are \$2.29/Mcf.

Table VI

Comparative Statics: Variation in the Capital Costs

Values reported are the expected profits for the Troll gas field calculated using the full set of input data displayed in Table V, except for the capital cost, which is varied as displayed here.

Capital Cost: (\$/Mcf)	0.69	0.89	1.09	1.29	1.49	1.69	1.89	2.09	2.29
Forward Sales (\$ million)	6880	6385	5478	4571	3664	3709	2885	2061	2447
Spot Sales (\$ million)	6791	5849	4907	3965	3023	2080	1138	196	0
Difference (F) (\$ million)	89	536	571	606	641	1629	1747	1865	2447
% Change	-1	-8	-10	-13	-17	-44	-61	-90	-100

III. Conclusion

Recent work in the economics of information has emphasized the role of long-term contracts in mitigating strategic problems which arise in imperfect or incomplete markets. In this paper, we have shown that an auction model can be used to operationalize the results of this body of literature. Using the auction model, we can estimate the significance of these strategic problems for a given project and the portion of the project's value that is secured to the producer by means of long-term contracts. This estimate allows us to incorporate strategic concerns into traditional and practical project valuation problems.

The strategic problems discussed in this paper are relevant for commodities which are traded in markets that are not perfectly competitive and for which large-scale capital investments are necessary. The imperfect competition may be created in some cases when the capital investment is dedicated to a small set of buyers/sellers in an industry that otherwise includes a large number of buyers/sellers.

The strategic value to long-term contracts arises because the equilibrium prices negotiated in forward and spot contracts are influenced differently by the large-scale capital investments made by the supplier. Use of forward contracts allows the capital investments to be made contingent upon the results of the negotiations. Spot contracts, on the other hand, are negotiated after capital has been irretrievably invested. This difference impacts on the relative expected profits of forward and spot sales through two factors. First, the distribution of spot prices is typically biased downward due to the loss of bargaining power; second, under forward negotiations no sale is made when the costs inclusive of capital charges would be greater than the negotiated price. The auction model yields a consistent estimate of the consequences of these two factors based upon fundamental data on the size of the market, the demands of the buyers, and the cost structure for the industry. Other models typically used for analyzing forward contracts, such as the arbitrage technique applied by Brennan and Schwartz (1985) to long-term coal contracts, do not incorporate this key influence on the relationship between anticipated forward and spot distributions.

We have applied the auction model to the analysis of long-term take-or-pay contracts used in the natural gas industry. While these long-term contracts have

been typical in this industry for decades, we show, for example, that, in the midwestern North American market, the growing number of buyers to which a given seller can route gas has significantly diminished the strategic value of these contracts. In contrast, where the number of available buyers remains small, as of gas routed to the New England market, the model yields a high strategic value to the traditional take-or-pay contract. In two cases from the European market, we show how the strategic value to the contract also depends significantly upon the cost structure of the gas fields, with the strategic value diminishing as the proportion of the costs that must be incurred prior to spot negotiations falls.

Two important avenues of further research immediately propose themselves. First, the structure of the environment analyzed in this paper is extremely stylized: there is only one seller, and the demand structures for all buyers are very simple and restrictive, as is the cost structure. On one hand, these factors make optimal the use of very simple sale contracts containing no contingency or flexibility and, therefore, allow us to view the problem with a minimum of complication. On the other hand, these restrictions limit the actual situations for which our model will yield accurate results. Moreover, these restrictions prevent us from making any analysis of or recommendations concerning the optimal design of more complicated forward contracts. We are currently analyzing the application of models of negotiations and bargaining in more complicated environments to the problem of long-term contracting.

The second area of further research follows from the fact that we have focused upon estimating the strategic factors that motivate the use of long-term contracts. We have left out of the analysis, for example, an estimation of the costs imposed upon both parties by the use of a forward contract. If, for example, production decisions should be made contingent upon information that arrives after the capacity must be installed, and if the forward contract cannot easily incorporate this contingency because the information is private to one party, then a forward contract will have strategic costs relative to the use of the spot market, and these costs may outweigh the strategic value analyzed in this paper. The problem is the subject of our current research. The key choices that we are analyzing are the structure of optimal price indexes and the optimal design of interim "take" or quantity decision rules.

Appendix 1: Definition of Values Used in the Modified Vickrey Auction

In accordance with Harris and Raviv (1981), to define the variances (A_1, \dots, A_{l-1}) we first define three component variables:

$$a_i = \left(\frac{1}{k}\right)^{N-1} \sum_{j=Q}^{N-1} \sum_{l=0}^{Q-2} \binom{N-1}{j} \binom{j}{l} (k-i)^l (i-1)^{N-j-1}, \quad i = 1, \dots, k-1,$$

$$b_i = \left(\frac{1}{k}\right)^{N-1} \binom{N-1}{N-Q} (k-i)^{Q-1} [i^{N-Q} - (i-1)^{N-Q}], \quad i = 1, \dots, k-1,$$

$$c_i = \left(\frac{1}{k}\right)^{N-1} \sum_{j=Q}^{N-1} \sum_{l=0}^{Q-1} \binom{N-1}{j} \binom{j}{l} (k-i)^l (i-1)^{N-j-1} \frac{Q-l}{j-l-1}, \quad i = 1, \dots, k-1,$$

which may then be combined as

$$A_i = 1 - \frac{c_i - a_i}{b_i}.$$

Appendix 2: Sample Displays of Auction Results and Calculation of Γ

In this section, we present three examples of the modified Vickrey auction. The first example illustrates the rules of the auction with a complete listing of the price and quantity allocations for every combination of buyer reservation values in the feasible event space and the calculation of the expected profit. The second and third examples illustrate our calculation for the value of forward contracting. In the second example, a complete listing of price and quantity allocations from an auction with the forward contracting minimum bid is displayed. In the third example, a complete listing is given for identical parameters, but for an auction with the spot contracting minimum bid. The analysis of these two cases explicates the proof that the value to forward contracting is positive.

Example 1: Table VII displays the full set of possible outcomes for an auction in which the manufacturer can produce a maximum of one unit at marginal fixed cost $k = 0.5$ and marginal variable cost $v = 0$, in which there are three potential buyers, each with possible reservation values $(R_1, \dots, R_4) = (0, 1, 2, 3)$. The first column of the table indexes the possible events; the second column containing the vector of four numbers is a list of each possible combination of numbers of buyers at each of the four reservation prices; the third column is the frequency of that event given our assumed uniform distribution for each reservation value over the range of reservation prices; the fourth column lists the modified Vickrey auction price that would follow for each event; the fifth column lists the total quantity that would be sold; the sixth column lists the producer’s total profits for each event. Displayed at the bottom is the expected profit to the producer from this auction sale of forward contracts.

Referring to the display in Table VII we will discuss several different possible outcomes for the set of buyer reservation values as a tool for explicating the properties of the auction model and its relation to the likely outcomes from contract negotiations:

Case 1: In the first event, there is no buyer with a reservation value greater than zero; the producer will not agree to install capacity for a price less than the total marginal cost, 0.5, and, therefore, the quantity sold is zero.

Case 2: In the second, third, and fourth events, there is only one buyer with a reservation value greater than or equal to the total marginal cost, and, therefore, there is no competition driving the price above the minimum; as stated in equation (6), when this is the case, the price is set at the minimum reservation price above the marginal cost, and, therefore, the modified Vickrey price is one.

Case 3: In the fifth event, there are two buyers with reservation values equal to one, and the price is therefore competed up to one, but the producer cannot charge a higher price.

Case 4: In the sixth event, there is one buyer with a reservation value equal

Table VII

The Outcomes from a Modified Vickrey Auction

The results of the auction are calculated using the following input parameters corresponding to Example 1 in Appendix 2: there are three buyers, each of whose reservation values may range over the set {0, 1, 2, 3}; the probability that any single buyer's reservation value will equal any one of the elements of the set of possible values is defined by the uniform distribution; the seller's maximum capacity of production is one unit, the operating costs are zero, and the capital costs of installing the maximum capacity are 0.5. The set of events is defined by the total number of buyers with each of the possible four reservation values. For example, in event 1 there are three buyers with reservation values equal to zero. In event 5 there is one buyer with a reservation value equal to zero and two buyers with reservation values equal to one.

The Space of Events						Outcomes of the Modified Vickrey Auction		
Index	Number of Buyers with Reservation Values				Probability of Event	Price ^a	Quantity Sold ^b	Profit ^c
	0	1	2	3				
1	3	0	0	0	0.01563	1.0	0	0.0
2	2	1	0	0	0.04688	1.0	1	0.5
3	2	0	1	0	0.04688	1.0	1	0.5
4	2	0	0	1	0.04688	1.0	1	0.5
5	1	2	0	0	0.04688	1.0	1	0.5
6	1	1	1	0	0.09375	1.6	1	1.1
7	1	1	0	1	0.09375	1.6	1	1.1
8	1	0	2	0	0.04688	2.0	1	1.5
9	1	0	1	1	0.09375	2.5	1	2.0
10	1	0	0	2	0.04688	3.0	1	2.5
11	0	3	0	0	0.01563	1.0	1	0.5
12	0	2	1	0	0.04688	1.6	1	1.1
13	0	2	0	1	0.04688	1.6	1	1.1
14	0	1	2	0	0.04688	2.0	1	1.5
15	0	1	1	1	0.09375	2.5	1	2.0
16	0	1	0	2	0.04688	3.0	1	2.5
17	0	0	3	0	0.01563	2.0	1	1.5
18	0	0	2	1	0.04688	2.5	1	2.0
19	0	0	1	2	0.04688	3.0	1	2.5
20	0	0	0	3	0.01563	3.0	1	2.5

Expected Profits = 1.43^d

^a Calculated using equations (3) and (4).

^b The sum of the quantity sold to each buyer as calculated from equation (2).

^c Calculated using equation (6).

^d Calculated using equation (7).

to two and one with a reservation value equal to one; competition in this event will always drive the price at least to one. The buyer with the higher reservation value may be forced in some cases to bid a price greater than one in order to obtain the unit of supply, and, therefore, for this event the expected price in the modified Vickrey auction is set slightly above one.

Case 5: In the eighth event, there are two buyers with reservation values of two, and they therefore compete the price up to two.

Example 2: Table VIII displays the full set of possible outcomes for an auction in which the manufacturer can produce a maximum of two units at marginal

Table VIII
Auction Outcomes for Forward Sales

The results of the auction for forward sales are calculated using the following input parameters corresponding to Example 2 in Appendix 2: there are three buyers, each of whose reservation values may range over the set {0, 1, 2, 3}; the probability that any single buyer's reservation value will equal any one of the elements of the set of possible values is defined by the uniform distribution; the seller's maximum capacity of production is two units, the operating costs are zero, and the capital costs of installing the maximum capacity are 0.5. The set of events is defined by the total number of buyers with each of the possible four reservation values. For example, in event 1 there are three buyers with reservation values equal to zero. In event 5 there is one buyer with a reservation value equal to zero and two buyers with reservation values equal to one.

The Space of Events					Outcomes of the Modified Vickrey Auction			
Index	Number of Buyers with Reservation Values				Probability of Event	Price ^a	Quantity Sold ^b	Profit ^c
	0	1	2	3				
1	3	0	0	0	0.01563	1.0	0	0.0
2	2	1	0	0	0.04688	1.0	1	0.5
3	2	0	1	0	0.04688	1.0	1	0.5
4	2	0	0	1	0.04688	1.0	1	0.5
5	1	2	0	0	0.04688	1.0	2	1.0
6	1	1	1	0	0.09375	1.0	2	1.0
7	1	1	0	1	0.09375	1.0	2	1.0
8	1	0	2	0	0.04688	1.0	2	1.0
9	1	0	1	1	0.09375	1.0	2	1.0
10	1	0	0	2	0.04688	1.0	2	1.0
11	0	3	0	0	0.01563	1.0	2	1.0
12	0	2	1	0	0.04688	1.0	2	1.0
13	0	2	0	1	0.04688	1.0	2	1.0
14	0	1	2	0	0.04688	1.6	2	2.2
15	0	1	1	1	0.09375	1.6	2	2.2
16	0	1	0	2	0.04688	1.6	2	2.2
17	0	0	3	0	0.01563	2.0	2	3.0
18	0	0	2	1	0.04688	2.0	2	3.0
19	0	0	1	2	0.04688	2.7	2	4.4
20	0	0	0	3	0.01563	3.0	2	5.0

Expected Profits = 1.49^d

^a Calculated using equations (3) and (4).

^b The sum of the quantity sold to each buyer as calculated from equation (2).

^c Calculated using equation (6).

^d Calculated using equation (9).

fixed and variable costs $k = 0.5$ and $v = 0$, in which there are three potential buyers, each with possible reservation values $(R_1, \dots, R_4) = (0, 1, 2, 3)$.

Example 2 is therefore identical with example 1 except that the capacity of the producer has been expanded from one unit to two units. We can compare examples one and two to see how the relationship between the capacity and the number of buyers affects the outcome of the auction. Events numbered 7 and 8 in both Tables VII and VIII illustrate this relationship. In both of these events, the number of buyers with reservation values greater than marginal cost is two. In Table VII, the resultant prices for these two events were 1.6 and 2. This was due to competition between the two buyers for the one unit of output. In events 7 and 8 in Table VIII, although there are two buyers with reservation values above marginal cost, there are also two units of capacity available for sale, and, therefore, there is no competition driving the price up above the minimal reservation value greater than the cost of production, one.

Example 3: Table IX displays the results for this example with $Q = 2$ under the assumption that the manufacturer installs the capacity and sells the output spot. In Table IX, the sixth column lists the operating profits for each event, i.e., the profits from the spot sale when the sunk capital costs are disregarded. In the seventh column, the fixed costs are allocated to each event, and the net profits for each event are calculated. The total expected profit is displayed at the bottom.

A comparison of the results from Table VIII with the results from Table IX illustrates the difference in profits that a seller can anticipate from using forward versus spot contracts. For example, in events 2-4, there is only one buyer with a valuation above the marginal cost, $k - v$. When the firm is negotiating forward contracts, i.e., as displayed in Table VIII, it sells only one unit and incurs the capital cost only for the installation of one unit of capacity. When the firm calculates the expected results from installing the capacity and negotiating spot sales, i.e., as displayed in Table IX, the firm has already incurred the capital cost of installing one unit of capacity and cannot in these cases earn a price which covers the fixed costs. In events 5-10, there are two buyers with valuations greater than the marginal costs of production, inclusive of capital cost. When the firm is negotiating forward contracts, i.e., as displayed in Table VIII, it installs the capacity to sell to these buyers only because the buyers agree to a price greater than the costs of production. When the firm calculates the expected results from installing the capacity and negotiating spot sales, i.e., as displayed in Table IX, it has installed capacity and therefore cannot force the price up to cover the marginal costs of production; the price is determined exclusively by competition, and, in these events, competition does not drive the price above the costs of production. In the remaining events, the price is determined exclusively by competition both for forward contracting and for spot contracting, and, therefore, the results are identical for the two cases.

In Table IX, the expected profits from selling spot are calculated and then compared with the expected profits from selling forward as exhibited in Table VIII. In this example, the expected profits from the operations decline by more than 38 percent from 1.49 to 0.92, when the producer fails to secure the forward contracts for its output prior to incurring the capital costs.

Table IX
Auction Outcomes from Spot Sales

The results of the auction for spot sales are calculated using the following input parameters corresponding to Example 3 in Appendix 2: there are three buyers, each of whose reservation values may range over the set {0, 1, 2, 3}; the probability that any single buyer's reservation value will equal any one of the elements of the set of possible values is defined by the uniform distribution; the seller's maximum capacity of production is two units, the operating costs are zero, and the capital costs of installing the maximum capacity are 0.5. The set of events is defined by the total number of buyers with each of the possible four reservation values. For example, in event 1 there are three buyers with reservation values equal to zero. In event 5 there is one buyer with a reservation value equal to zero and two buyers with reservation values equal to one.

The Space of Events					Outcomes of the Modified Vickrey Auction				
Index	Number of Buyers with Reservation Values				Probability of Event	Price ^a	Quantity Sold ^b	Operating Profit ^c	Total Profit ^d
	0	1	2	3					
1	3	0	0	0	0.01563	0.0	2	0.0	-1.0
2	2	1	0	0	0.04688	0.0	2	0.0	-1.0
3	2	0	1	0	0.04688	0.0	2	0.0	-1.0
4	2	0	0	1	0.04688	0.0	2	0.0	-1.0
5	1	2	0	0	0.04688	0.6	2	1.2	0.2
6	1	1	1	0	0.09375	0.6	2	1.2	0.2
7	1	1	0	1	0.09375	0.6	2	1.2	0.2
8	1	0	2	0	0.04688	0.6	2	1.2	0.2
9	1	0	1	1	0.09375	0.6	2	1.2	0.2
10	1	0	0	2	0.04688	0.6	2	1.2	0.2
11	0	3	0	0	0.01563	1.0	2	2.0	1.0
12	0	2	1	0	0.04688	1.0	2	2.0	1.0
13	0	2	0	1	0.04688	1.0	2	2.0	1.0
14	0	1	2	0	0.04688	1.6	2	3.2	2.2
15	0	1	1	1	0.09375	1.6	2	3.2	2.2
16	0	1	0	2	0.04688	1.6	2	3.2	2.2
17	0	0	3	0	0.01563	2.0	2	4.0	3.0
18	0	0	2	1	0.04688	2.0	2	4.0	3.0
19	0	0	1	2	0.04688	2.7	2	5.4	4.4
20	0	0	0	3	0.01563	3.0	2	6.0	5.0
Expected Operating Profits								=	1.92
Expected Total Profits								=	0.92
Loss Relative to Expected Profits from Forward Sales, Γ								=	0.57
Percent Change in Profits Relative to Forward Sales								=	-38%

^a Calculated using equations (3) and (5).

^b The sum of the quantity sold to each buyer as calculated from equation (2).

^c Calculated using equation (6) with the capital costs added back in.

^d Calculated using equation (6).

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